

M.Sc. (MATHEMATICS)

ASSIGNMENT

Session 2025-2027 (II-Semester)

&

Session 2024-2026 (IV-Semester)



CENTRE FOR DISTANCE AND ONLINE EDUCATION

**GURU JAMBHESHWAR UNIVERSITY OF SCIENCE &
TECHNOLOGY
HISAR, HARYANA-1250001.**

Compiled & Prepared by

Mohit Kumar

Assistant Professor & Programme Coordinator M.Sc. Mathematics
Centre For Distance And Online Education, GJUS&T Hisar.

Email:- babamohithhh@gmail.com

Programme: M.Sc. (Mathematics) Semester:-IV

Important Instructions

- (i) Attempt all questions from the each assignment given below. Each question carries 05 marks and the total marks are 15.
- (ii) All questions are to be attempted in legible handwriting on plane white A-4 size paper and to be uploaded online to the Directorate of Distance Education for evaluation.

Nomenclature of Paper: Functional Analysis

Paper Code: MAL-641

Total Marks = 15 + 15

ASSIGNMENT-I

Q.1. State and prove Riesz-Fisher Theorem.

Q.2. State and prove Open Mapping Theorem.

Q.3. Let M be a closed linear subspace of a Normed linear space N . If the norm of coset $x + M$ in the quotient space $\frac{N}{M}$ is defined by

$$\|x + M\| = \inf. \{ \|x + m\|; m \in M \}.$$

Then $\frac{N}{M}$ is a normed linear space.

ASSIGNMENT-II

Q.1. State and prove Minkowski's Inequality.

Q.2. Prove that if a normed linear space X is reflexive, then X^* is also reflexive

Q.3. State and prove Riesz-Representation Theorem for Hilbert spaces.

Nomenclature of Paper: Differential Geometry

Paper Code: MAL-642

Total Marks = 15 + 15

ASSIGNMENT-I

- Q.1.** (a) Established **Serret Frenet formulae** $\mathbf{t}' = k \mathbf{n}$, $\mathbf{n}' = \tau \mathbf{b} - k \mathbf{t}$, $\mathbf{b}' = -\tau \mathbf{n}$ where the symbols have their usual meaning.
 (b) For the curve $x = 3t, y = 3t^2, z = 2t^3$, show that any plane meets it in three points and deduce the equation to the osculating plane at $t = t_1$.

- Q.2. (a)** If C is a curve for which \mathbf{b} varies differentially with arc length. Then to show that a necessary and sufficient condition that C is a plane curve is that $\tau = 0$ at all points.
 (b) Let C be a curve given by the equation $\mathbf{r} = (u, u^2, u^3)$, find the curvature and torsion of C at the point (0,0,0). Also, find the equation of its binormal line and normal plane at the point (1,1,1).

- Q.1.** Given the curve $\mathbf{r} = (e^{-u} \sin u, e^{-u} \cos u, e^{-u})$. Find at any point 'u' of this curve
 (i) Unit tangent vector \mathbf{t}
 (ii) The equation of tangent
 (iii) The equation of normal plane
 (iv) The curvature
 (v) The unit principal normal vector \mathbf{b} , and
 (vi) The equation of the binormal.

ASSIGNMENT 1I

- Q.1(a)** Find the envelope of the plane $3xt^2 - 3yt + z = t^3$ and show that its edge of regression is the curve of the intersection of the surfaces $y^2 = zx$, $xy = z$.
 (b) Find the principal curvatures etc. on the surface generated by the binormals of a twisted curve.
Q.2.(a) Find the principal curvatures and the lines of curvature on the right helicoids $x = u \cos \phi$, $y = u \sin \phi$, $z = c\phi$.
 (b) Find the envelope of the plane $(x/a) \cos \theta \sin \phi + (y/b) \sin \theta \sin \phi + (z/c) \cos \phi = 1$.
Q.3.(a) To prove that the envelope of a developable plane whose equation involves one parameter is a developable surface
 (b) A necessary and sufficient condition that a curve on a surface be a line of curvature is that the surface normal along the curve is developable.

Nomenclature of Paper: Mechanics of Solid-II

Paper Code: MAL-643

Total Marks = 15 + 15

ASSIGNMENT-I

- Q.1.** Derive the formulae for stresses in terms of two analytic functions, assuming plane strain conditions.
Q.2. Discuss the problem of deflection of a central line of an elastic beam by transverse load.
Q.3. Derive constitutive equation for a Maxwell material. Also discuss its creep and

relaxation phases.

ASSIGNMENT-II

- Q.1.** Find torsional moment in the problem of torsion of an elliptic cylinder.
- Q.2.** Obtain the frequency equation for Rayleigh waves. Also show that these are non-dispersive and particle motion is elliptic retrograde.
- Q.3.** Solve the problem of a long thick-walled tube in plane strain whose material is elastic in dilatation and Maxwell viscoelastic in distortion with internal pressure p and outer surface is in contact with a rigid body.

Nomenclature of Paper: Integral Equation

Paper Code: MAL-644

Total Marks = 15 + 15

ASSIGNMENT-I

- Q.1.** Find the integral equation corresponding to boundary value problem (B.V.P.)

$$y''(x) + \lambda y(x) = 0, \quad y(0) = 0, y(1) = 1.$$

- Q.2.** State and prove Green's formula.

- Q.3.** Solve the integral equation: $y(x) = x + \lambda \int_0^\pi \sin(x) \sin(t) y(t) dt$

ASSIGNMENT-II

- Q.1.** Find the resolvent kernel of Volterra Integral Equation with kernel $K(x, t) = \frac{\cosh t}{\sinh t}$

- Q.2.** Transform the problem: $y''(x) + y = x, y(0) = 1, y'(1) = 0$
to Fredholm integral equation.

- Q.3.** State and prove Fredholm's Third Theorem.

Nomenclature of Paper: Advanced Fluid Mechanics

Paper Code: MAL-645

Total Marks = 15 + 15

ASSIGNMENT-I

- Q.1.** Discuss the properties of boundary layer equations.
- Q.2.** Obtain the equation of motion of a gas.

Q.3. Obtain the principal stresses and principal stress direction if the stress tensor at a point is given by

$$\tau_{ij} = \begin{pmatrix} 6 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

ASSIGNMENT-II

Q.1. Determine the local frictional coefficient for flow over a flat plate, based on Karman integral equation.

Q.2. Derive Navier-Stokes's equation of motions in Cartesian coordinates.

Q.3. Define Reynold Number, Froude number, Mach number and Eckert number.

1 Nomenclature of Paper: Computing Lab-3

Paper Code: MAP-648 Total Marks = 15 + 15

1.1 ASSIGNMENT-I

Q.1. System of equations: (5)

$$\sin nz = nz \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2}\right) \nabla \cdot \vec{q} = 0$$
$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + v \frac{\partial^2 T}{\partial y^2} \right)$$

Q.2 Write syntax for the following the matrix: (5)

$$\tau_{ij} = \begin{pmatrix} 6 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

Q.3. Discuss the commands that can be use to write multiple equations. (5)

1.2 ASSIGNMENT-II

Q.1. What is use of multiline-environment, show by an example. How IEEE eqnarray – environment is used and what are the advantages. (5)

Q.2. Syntax for the piecewise function: (5)

$$P_A(x) = \begin{cases} 1 & \text{if } x = 0 \\ 2 & \text{if } x = 1 \\ 4 & \text{if } x = -1 \end{cases}$$

Q.3. Table using tabular: (5)

| X | Y | Z | |
|----------|----------|----------|---|
| A | C_1 | a | b |
| | C_2 | c | d |
| B | C_3 | e | f |
| | | f | h |

Programme: M.Sc. (Mathematics) Semester:-II

Important Instructions

- (i) Attempt all questions from each assignment given below. Each question carries marks mentioned in brace and the total marks are 15.
- (ii) All questions are to be attempted in legible handwriting on plane white A-4 size paper and to be uploaded online to the Directorate of Distance Education for evaluation.

Nomenclature of Paper: Abstract Algebra

Paper Code: MAL-521

Total Marks = 15 + 15

Assignment-I

1. Let G be a finitely generated abelian group. Prove that G can be decomposed as a direct sum of a finite number of cyclic groups C_i , i.e.,

$$G = C_1 \oplus C_2 \oplus \cdots \oplus C_t$$

where either all C_i 's are infinite or for some $j < k$, C_1, C_2, \dots, C_j are of order m_1, m_2, \dots, m_j , respectively, with $m_1 \mid m_2 \mid \cdots \mid m_j$, and the rest of C_i 's are infinite. (5)

2. Define nilpotent transformation with a suitable example. Also, prove that all the characteristic roots of a nilpotent transformation are zero. If $T \in A(V)$ has all its characteristic roots in F , then there exists a basis of V in which the matrix of T is triangular. (5)

3. Let V be a vector space over F and $T \in A(V)$. If the minimal polynomial of T over F is

$$f(x) = a_0 + a_1x + \cdots + a_{m-1}x^{m-1} + x^m$$

and V is a cyclic $F[x]$ -module, then prove that there exists a basis of V under which the matrix of T is the companion matrix of $f(x)$. (5)

Assignment-II

1. Let M be an R -module. Prove that the following conditions are equivalent: (5)
 - (i) M is Noetherian.
 - (ii) Every non-empty family of submodules of M has a maximal element.
 - (iii) Every submodule of M is finitely generated.
2. Show that if R is a Noetherian ring with identity, then $R[x]$ is also a Noetherian ring. (5)
3. Define similar transformation and prove that if a subspace $W \subset V$ is invariant under T , then T induces a linear transformation \bar{T} on V/W , defined by:

$$(v + W)\bar{T} = vT + W$$

Further, if T satisfies a polynomial $q(x)$ over F , then so does \bar{T} . (5)

Nomenclature of Paper: Measure & Integration Theory

Paper Code: MAL-522

Total Marks = 15 + 15

Assignment-I

1. Prove that the characteristic function χ_A is measurable if and only if A is a measurable set. (5)
2. State and prove the **Bounded Convergence Theorem**. (5)
3. State and prove **Lusin's Theorem**. (5)

Assignment-II

1. State and prove **Egoroff's Theorem**. (5)
2. Answer the following questions:
 - (i) Differentiate between Lebesgue and Riemann integration.
 - (ii) What are measurable functions? Give an example.

- (iii) What are convex functions?
- (iv) Explain L^p -spaces with a suitable example.
- (v) What are functions of bounded variation? (5)

3. Prove the following properties of integrable functions over a measurable set E :

- (i) If f, g are integrable over E , then $f + g$ is integrable and:

$$\int_E (f + g) = \int_E f + \int_E g$$

- (ii) If $f \leq g$ almost everywhere, then:

$$\int_E f \leq \int_E g$$

- (iii) If A and B are disjoint measurable subsets of E , then:

$$\int_{A \cup B} f = \int_A f + \int_B f$$

(5)

Nomenclature of Paper: Method of Applied Mathematics

Paper Code: MAL-523

Total Marks = 15 + 15

Assignment-I

1. Let the random variable X have the **exponential distribution** with probability density function

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0, \lambda > 0.$$

Find the **mean**, **variance**, and **moment generating function (MGF)** of the distribution. (5)

2. Define the **normal distribution**. Show that the moment generating function (MGF) of a normal random variable with mean μ and variance σ^2 is

$$M_X(t) = \exp \left(\mu t + \frac{1}{2} \sigma^2 t^2 \right).$$

Hence, deduce the **first** and **second moments** of the distribution. (5)

3. If a random variable X follows a **binomial distribution** with parameters n and p , and it is given that

$$P(X = 1) = 3P(X = 2), \quad P(X = 2) = 3P(X = 3),$$

find the values of n and p . Also, compute the **mean** and **variance** of the distribution.
(5)

Assignment-II

1. Represent the vector

$$\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$$

in spherical coordinates (r, θ, ϕ) . (5)

2. Use the method of Fourier transforms to determine the displacement $u(x, t)$ of an infinite string, given that the string is initially at rest and that the initial displacement is $f(x)$, where $-\infty < x < \infty$. (5)

3. Find the Fourier sine transform of $f(t) = e^{-at}$, where $a > 0$. Hence, deduce the Fourier cosine transform of

$$f(t) = \frac{t}{a^2 + t^2}. \quad (5)$$

Nomenclature of Paper: Ordinary Differential Equations-II

Paper Code: MAL-524

Total Marks = 15 + 15

Assignment-I

1. Solve the following system of linear differential equations using the **eigenvalue method**:

$$\begin{cases} \frac{dx}{dt} = 4x + y \\ \frac{dy}{dt} = -2x + y \end{cases}$$

Find the general solution. (5)

2. State and prove Abel Liouville's formula. (5)

3. Determine the nature of the critical point $(0, 0)$ of the system

$$\frac{dx}{dt} = 2x - 7y, \quad \frac{dy}{dt} = 3x - 8y,$$

and determine whether or not the point is stable. (5)

Assignment-II

1. Use the calculus of variation to find the curve joining points $(0, 0, 0)$ and $(1, 2, 4)$ of shortest length. Also, find the distance between these two points. (5)

2. Analyze the following nonlinear system:

$$\begin{cases} \frac{dx}{dt} = x(1 - y) \\ \frac{dy}{dt} = y(x - 1) \end{cases}$$

(a) Find the equilibrium points.

(b) Linearize the system at each equilibrium point.

(c) Classify the stability of each equilibrium. (5)

3. Using the **Euler–Lagrange equation**, find the function $y(x)$ that minimizes the functional:

$$J[y] = \int_0^1 (y'^2 + y^2) dx, \quad \text{with } y(0) = 0, \quad y(1) = 1$$

(5)

Nomenclature of Paper: Complex Analysis-II

Paper Code: MAL-525

Total Marks = 15 + 15

Assignment-I

1. State and prove the Weierstrass factorization theorem. (5)

2. State and prove Jensen's formula. (5)

3. Prove that if $|z| \leq 1$ and $p \geq 0$, then

$$|1 - E_p(z)| \leq |z|^{p+1}.$$

(5)

Assignment-II

1. State and prove Hurwitz's theorem. (5)
2. State and prove the Riemann mapping theorem. (5)
3. (i) State Hadamard's factorization theorem.
(ii) Show that

$$\sin(\pi z) = \pi z \prod_{n=1}^{\infty} \left(1 - \frac{z}{n}\right)$$

by Hadamard's factorization theorem. (5)

Nomenclature of Paper: Advanced Numerical Method

Paper Code: MAL-526

Total Marks = 15 + 15

Assignment-I

1. Approximate the following improper integral using a suitable numerical integration method:

$$\int_1^{\infty} \frac{1}{x^2 + 1} dx$$

(5)

2. Using a second order method with $h = \frac{1}{2}$, find the solution of the BVP

$$(1 + x^2)y'' + 2xy' - y = 1 + x^2, \quad y(0) = 0, \quad y'(1) = 1$$

(5)

3. The function $f(x, y)$ is known:

$$f(0, 0) = -1, \quad f(0, 1) = 2, \quad f(0, 2) = 3, \quad f(1, 0) = 4, \quad f(1, 1) = 0, \quad f(1, 2) = 4,$$
$$f(2, 0) = 2, \quad f(2, 1) = -2, \quad f(2, 2) = 3$$

For these values, construct the Newton's bivariate polynomial. Also, find the approximate values of $f(1.25, 0.75)$ and $f(1.0, 1.5)$. (5)

Assignment-II

1. Consider the first-order differential equation:

$$\frac{dy}{dx} = x + y, \quad y(0) = 1$$

Use the 4th-order Runge-Kutta method to solve the equation numerically in the interval $x \in [0, 1]$ with step size $h = 0.1$. (5)

2. Solve the following system of linear equations using the relaxation method (also known as the Successive Over-Relaxation method) with initial guess $(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}) = (0, 0, 0)$. Use a relaxation factor $\omega = 1.1$ and iterate until the absolute difference between successive values is less than 0.0001:

$$\begin{aligned} 10x_1 + 2x_2 + x_3 &= 92 \\ x_1 + 20x_2 - 2x_3 &= -44 \\ -2x_1 + 3x_2 + 10x_3 &= 22 \end{aligned} \tag{5}$$

3. The following values of x and y are given:

| | | | | |
|-----|---|---|---|----|
| x | 1 | 2 | 3 | 4 |
| y | 1 | 2 | 5 | 11 |

Find the cubic splines and evaluate $y(1.5)$ and $y'(3)$. (5)

Nomenclature of Paper: Computing Lab-Matlab

Paper Code: MAL-527

Total Marks = 15 + 15

Assignment-I

1. Write a program to calculate mean and median. (5)
2. Write a program to find the adjoint of a matrix. (5)
3. Write a program to operate arithmetic operators on vector. (5)

Assignment-II

1. Write a program to draw multiple graphs on same plot. (5)
2. How to solve systems of linear equations using matrices in MATLAB. (5)
3. Write a program to operate element-wise operations on matrices. (5)